Problem 1 Find the domain of the following function:

$$f(x) = \frac{1}{\sqrt{x-3}} \cdot \frac{x^2+1}{x^2-25}$$

Additional problem(s): Find the domain of f in these cases:

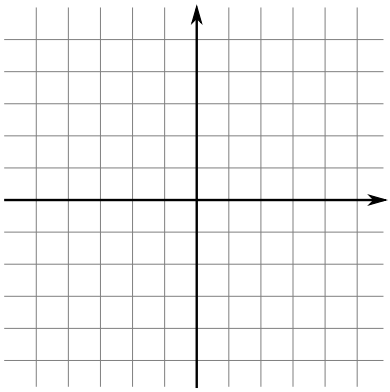
- $f(x) = \dfrac{1}{\sqrt{|x-3|-7}}$ (

  Solution: $(-\infty, -4) \cup (7, \infty))$
- $f(x) = \dfrac{1}{x^2+1} \cdot \dfrac{\sqrt{9-x^2}}{x^2-4}$
- $f(x) = \sin\left(\dfrac{\log\left(\sqrt{3-x}\right)}{3+x}\right)$

Problem 2

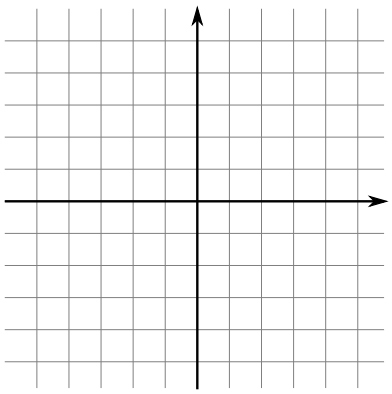
a. Sketch the graph of the following piecewise-defined function:

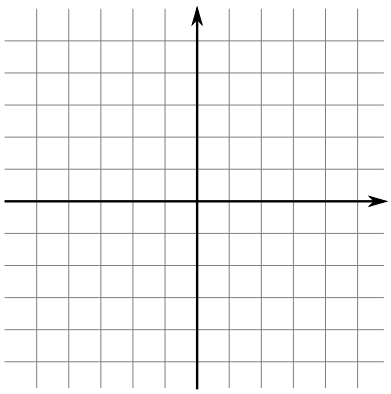
$$f(x) = \begin{cases} \sqrt{x}, & \text{if } x < 4; \\ x - 1, & \text{if } x \geq 4. \end{cases}$$



b. Is this function continuous?

c. Sketch the graph of $2f(x-2)$ on the left [**4 points**], and graph of $f^{-1}(x)$ on the right





Problem 3 You and your friend attended a show of the famous band Klein Four. After the show ended, you bought 7 records, and your friend bought 4. Your total expenses were \$125, and your friend's expenses totaled \$80.

The ticket prices are the same, and all records cost the same. Build a model of the cost of attendance of the show. Interpret the slope and y-intercept of the function you get.

Problem 4 Let f, g be functions whose domain is $(-\infty, +\infty)$. The values of f and g for some values of x are given in the table below:

x	1	2	3	4	5
f(x)	3	3	3	3	2
g(x)	1	6	10	5	3

a. What is $f \circ g(5)$?

b. What is $g \circ f(5)$?

c. Is f one-to-one?

d. Is g one-to-one?

e. **Additional problem(s):**

- Let h be a function whose domain is $\{1, 2, 3, 4, 5\}$, and $h(x) = g(x)$ wherever defined. Is h one-to-one?

Problem 5 The population of fruit flies around the bananas in your kitchen (that you totally forgot about) doubles every 36 hours. There are 10 flies buzzing happily now.

a. How many fruit flies will you have in three days?

b. What is the function $f(x)$ that describes the number of flies you have after x hours?

Problem 6 Constant hyperinflation in the country of Artztozka decreases the value of savings in Artztozkan Grubles (as measured in USD) by a factor of 0.8 every 18 months. You just purchased \$1000 worth of Arztztozkan Grubles.

a. What will be the value of your investment in 2 years?

b. Arztotzkan (never-changing) president swore to resign if an investment into Grubles depreciates to 10% of its initial value in USD. How many years will pass before you are able to call him out on that fake promise?

Problem 7 You measure velocity of a falling water balloon, in m/s, in terms of time passed, in s, since you dropped it from the rooftop of the mathdepartment towards your favorite math professor.

t, s	1	1.5	2	2.5	3
V, m/s	14	20	23	28	35

It seems like the constant pull of gravity yields a linear model. Use linear regression to determine the model.

Problem 8 The x coordinate of the the tip of the second clock hand, in cm, is given by the equation $x(t) = 15\sin(2\pi t/60)$.

a. Find the average velocity V_{ave} on the following intervals I:

1. $I = [30, 32]$.

   Solution: $V_{\text{ave}} = \dfrac{x(32) - x(30)}{2} = ...$

2. $I = [30, 30.5]$, $V_{\text{ave}} = ...$

3. $I = [30, 30.1]$, $V_{\text{ave}} = ...$

4. $I = [30, 30.01]$

b. What the instantaneous horizontal velocity at $t = 30$s?

Problem 9 Let $f(x) = x^2$. Calculate (or approximate to 4 decimal places) the following limit:

$$\lim_{h\to 0} \frac{f(a+h) - f(a)}{h}$$

for $a = 7$.

Problem 10 Calculate (or approximate to 4 decimal places) the following limit:

$$\lim_{x\to\infty} \frac{2x^3 + 5x + 1}{3x^4 + 5x^2 + 7}.$$

Problem 11 Let $f(x) = \sin(x)$, where x is in degrees, so $f(30) = 0.5$.

a. For the following intervals $I_a = (0.5 - a, 0.5 + a)$, find intervals J_b of the form $(30 - b, 30 + b)$ so that f takes values in I_a on the interval J_b you find.

1. $a = 0.02$, $I_a = (0.52, 0.48)$;

2. $a = 0.001$, $I_a = (0.501, 0.499)$;

b. Let $g(x)$ be defined as follows:

$$g(x) = \begin{cases} f(x), x \neq \dfrac{\pi}{4}; \\ 0, x = \dfrac{\pi}{4}. \end{cases} .$$

Find the limit $\lim\limits_{x\to 4} g(x)$, or state that it does not exist.