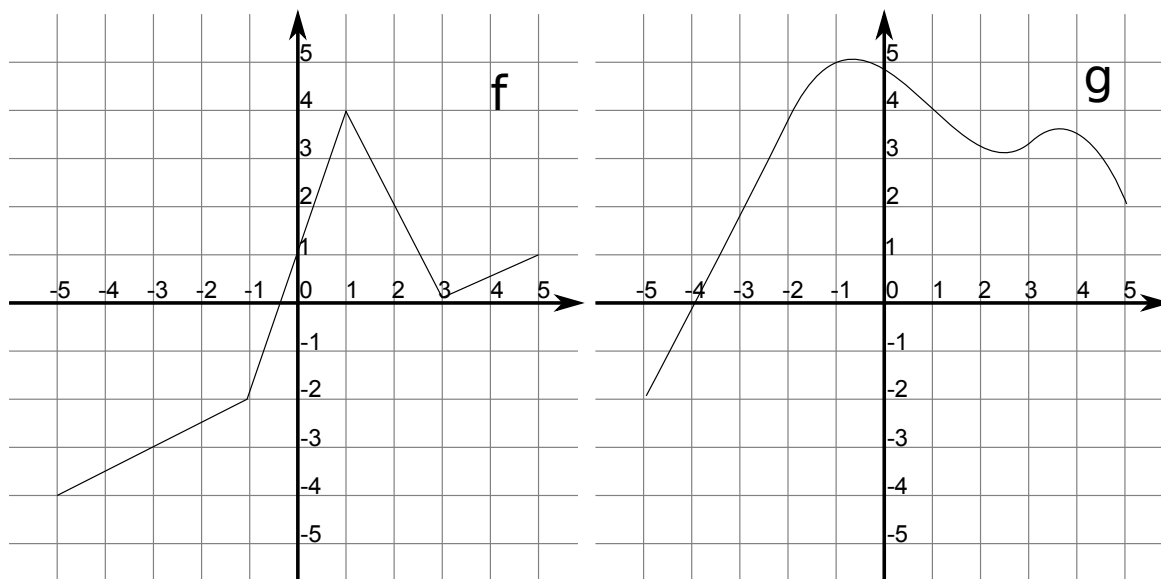


# MATH 131:501 Midterm Exam 2

## PRACTICE PROBLEMS

**NOTE:** The actual exam will differ in length and problem types, but solving these problems should leave you prepared.

**Problem 1:** Let  $f, g$  be functions whose graphs are provided below.



1. Where do  $f'$ ,  $g'$  exist?
2. Where is  $f(x)$  increasing/decreasing? Where is  $f'(x)$  positive/negative? Same questions for  $g$ .
3. Find  $f'(-3)$  and  $g'(-3)$
4. If  $F(x) = f(x)/g(x)$ ,  $F'(-3) =$
5. Estimate  $g'(1.5)$  from the graph. Use it to find the equation of the tangent line.
6. Sketch  $f'(x)$  [**3 points**] and  $g'(x)$  [**3 points**].

**Problem 2:** Let  $f(x) = \frac{1}{x+1}$ . Using the **definition of derivative only**, find the derivative of  $f(x)$ .

**Note:** show all work. No credit will be given for just an answer. **You may not use the power rule.**

**Problem 3:** Let  $f(x) = \ln(x)$ . Find  $F'(x)$  using **only** the chain rule and the fact that  $(e^x)' = e^x$ .

**Problem 4:** After taking an exam, you throw your favorite instructor through the window. The altitude of your instructor, in meters, as a function of time, in seconds, is given by the function  $f(t) = 6 - 5(t-1)^2$ . How fast (in the vertical direction) did you throw your favorite instructor? How fast is your instructor going at the moment of impact with the ground?

**Problem 5:** Let  $f, g : \mathbb{R} \rightarrow \mathbb{R}$  be differentiable functions. The values of  $f(x), f'(x), g(x), g'(x)$  for some values of  $x$  are given in the table below.

$x$	1	2	3	4	5	6
$f(x)$	2	3	5	4	6	2
$g(x)$	4	1	3	2	6	5
$f'(x)$	1	5	3	2	4	11
$g'(x)$	5	4	3	11	7	6

Using the rules of differentiation, compute the following. **Justify your work!**

1.  $F(x) = f(x) \cdot g(x)$ .  $F'(2) =$
2.  $F(x) = f(x)/g(x)$ .  $F'(2) =$
3.  $F(x) = g(f(x))$ .  $F'(3) =$
4.  $F(x) = e^{g(x)}$ .  $F'(5) =$
5.  $F(x) = 2f(x) + 3g(x)$ .  $F'(5) =$
6.  $F(x) = \ln(f(x) \cdot g(x))$ .  $F'(1) =$
7.  $F(x) = \frac{\sin(f(x))}{\cos(g(x) + 1)}$ .  $F'(3) =$ .

**Problem 6:** Find the following derivatives:

1.  $\frac{d}{dx} (x^{1/2} + x^{1/3} + 7) =$
2.  $\frac{d}{dx} \left( \frac{\cos(x)}{\sqrt{x^2 + 1}} \right) =$
3.  $\frac{d}{dx} \left( \frac{\sin(x^3)}{e^x} \right) =$
4.  $\frac{d}{dx} (\ln(\sin(x) \cos(x))) =$

**Problem 7:** Let  $f : \mathbb{R} \rightarrow \mathbb{R}$  be a differentiable function on the real line. Match the limits below to their values.

1.  $\lim_{a \rightarrow 0} \frac{f(x+a) - f(x)}{a}$

2.  $\lim_{a \rightarrow 0} \frac{f(2a) - f(-2a)}{4a}$

3.  $\lim_{a \rightarrow 0} \frac{f(a) - f(0)}{2a}$

4.  $\lim_{a \rightarrow 0} \frac{f(0) - f(2a)}{2a}$

5.  $\lim_{a \rightarrow 0} \frac{f(a) - f(0)}{a}$

6.  $\lim_{a \rightarrow 0} \frac{f(x+a) - f(x-a)}{2a}$

7.  $\lim_{a \rightarrow 0} \frac{f(-a) - f(a)}{-a}$

a.  $f'(0)$

b.  $-f'(0)$

c.  $f'(0)/2$

d.  $2f'(0)$

e.  $f'(x)$

f.  $-f'(x)$

g.  $f'(x)/2$

h.  $2f'(x)$