Compute the following integrals:

1. (3 points) This problem is done by integration by parts, where we integrate x and differntiate $\arctan x$:

$$\begin{aligned}\int x\arctan x\ dx &= \int \frac{x^2}{2}\arctan x - \int \frac{x^2}{2}\cdot\frac{1}{x^2+1}\ dx\\ &= \frac{x^2}{2}\arctan x - \frac{1}{2}\int \frac{x^2}{x^2+1}\ dx\\ &= \frac{x^2}{2}\arctan x - \frac{1}{2}\int 1-\frac{1}{x^2+1}\ dx\\ &= \frac{x^2}{2}\arctan x - \frac{1}{2}(x-\arctan x)+C\end{aligned}$$

2. (4 points) This is a regular trigonmetric integral problem. It can be done by writing $\cos^2(x) = 1-\sin^2(x)$ and a substitution $u=\sin(x)$:

$$\begin{aligned}\int \sin^4 x\cos^5 x\ dx &= \int \sin^4 x\left(\cos^2 x\right)^2\cos x\ dx\\ &= \int \sin^4 x\left(1-\sin^2 x\right)^2\cos x\ dx\\ &= \int u^4(1-u^2)^2\ du\\ &= \int u^4-2u^6+u^8\ du\\ &= \frac{u^5}{5}-\frac{2}{7}u^7+\frac{u^9}{9}+C\\ &= \frac{1}{5}\sin^5 x-\frac{2}{7}\sin^7 x+\frac{1}{9}\sin^9 x+C\end{aligned}$$

3. (3 points)

$$\int_{-1}^{1}\sqrt{1-x^2}dx$$

This problem requires no computation. Notice that the plot of $y=\sqrt{1-x^2}$ for $x = -1..1$ is a semicircle. Indeed, squaring both sides we get $y^2 = 1-x^2$ or $x^2+y^2=1$, equation for a circle of radius 1. See Figure 1.

The integral is half the area of the circle of radius 1 and equals $\pi/2$.

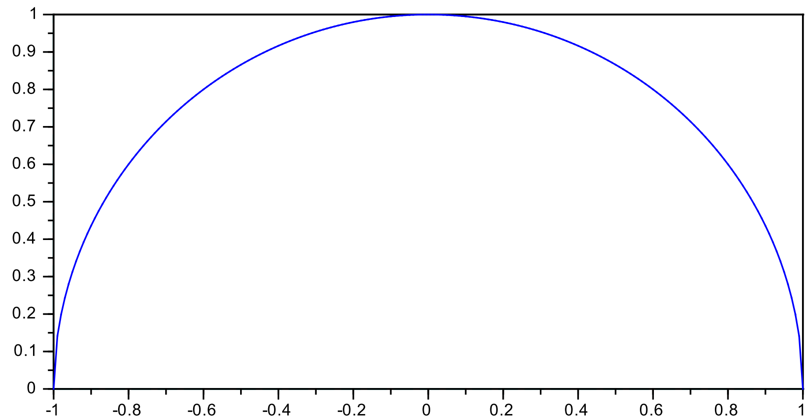

Figure 1: Plot of $y = \sqrt{1 - x^2}$.

Note: this problem **can** be done by trigonometric substitution with $x = \sin u$. This is how you would get the indefinite integral, or definite integral with arbitrary bounds. It was **not required** in this case if you understand what the integration does.

The idea of this problem is to avoid computation when you know what you are dealing with.