1. (3 points) Compute the following integral:

$$\int \frac{1}{(x-1)(x+1)}\,dx$$

Solution: Integration by partial fractions:

$$\begin{aligned}\frac{A}{x-1}+\frac{B}{x+1}&=\frac{1}{(x-1)(x+1)}\\ \frac{A(x+1)+B(x-1)}{(x-1)(x+1)}&=\frac{1}{(x-1)(x+1)}\\ A(x+1)+B(x-1)&=1\\ (A+B)x+(A-B)&=1.\end{aligned}$$

So we obtain the system:

$$\begin{cases}A+B &=0\\ A-B &=1\end{cases}$$

The solution is $A=1/2$, $B=-1/2$, so we have

$$\begin{aligned}\int \frac{1}{(x-1)(x+1)}\,dx &= \frac{1}{2}\int \frac{1}{x-1}+\frac{1}{x+1}\,dx\\ &=\frac{1}{2}(\ln(x-1)+\ln(x+1)).\end{aligned}$$

2. (3 points) Compute the following integral:

$$\int_1^3 \frac{1}{\sqrt{|x-2|}}\,dx$$

Solution: Perform a change of coordinates to make the integral look nicer: let $u=x-2$; then

$$\int_1^3 \frac{1}{\sqrt{|x-2|}}\,dx=\int_{-1}^1 \frac{1}{\sqrt{|u|}}\,du.$$

The integrad is undefined for $u=0$: there is a vertical asymptote, so we break the integral up. Also note that $|-u|=|u|$. Therefore,

$$\begin{aligned}\int_{-1}^1 \frac{1}{\sqrt{|u|}}\,du &= \int_{-1}^0 \frac{1}{\sqrt{|u|}}\,du+\int_0^1 \frac{1}{\sqrt{|u|}}\,du\\ &=2\int_0^1 \frac{1}{\sqrt{u}}\,du\\ &=2\cdot 2\cdot u^{1/2}|_0^1\\ &=4.\end{aligned}$$

3. (4 points) Find the length of the curve $y = \sqrt{1-x^2}$ from $x = -1/2$ to $x = 1/2$.

Solution: The arclength of the unit circle from $\frac{2}{3}\pi$ to $\frac{1}{3}\pi$ is $\frac{1}{3}\pi$.

Or, if you are feeling adventurous and like trigonometric integrals:

$$y = \sqrt{1-x^2} \Rightarrow y' = \frac{x}{\sqrt{1-x^2}}$$

$$\begin{aligned} L = \int_{-1/2}^{1/2} \sqrt{1+(y')^2}\, dx &= \int_{-1/2}^{1/2} \sqrt{1 + \frac{x^2}{1-x^2}}\, dx \\ &= \int_{-1/2}^{1/2} \sqrt{\frac{1}{1-x^2}}\, dx \\ &= \arcsin x \big|_{-1/2}^{1/2} \\ &= \pi/3. \end{aligned}$$