Homework 2 solutions

The following statement is false in general:

$\int_a^b f(x)dx$ is the area bounded by the graph of f, the x axis, and lines $x = a$ and $x = b$.

1. Provide an example of f, a, b for which the statement above is false.

   Solution: any function f that is negative somewhere on $[a, b]$, e.g.

   $$\int_0^{2\pi} \sin(x)dx = 0, \text{ but the area is nonzero.}$$

   It is interesting to consider functions that are unbounded on $[a, b]$. Is the area defined in such cases? What if the area is infinite? What if both the area and the integral diverge to positive infinity? Etc.

2. For a and b fixed, for which functions is the above statement true?

   Solution: this statement is true for all (integrable) functions f that are non-negative on $[a, b]$.