

Homework problem: In the old times, before cargo elevators were invented, the furniture was lifted into apartments by ropes through the windows. The walls of buildings were slightly slanted to allow for the rope to hang from the rooftop.

Calculate the work performed by lifting a 100kg piano to the 4-th floor window (10m above the ground) by a rope which is pulled from the roof top (20m above ground) and which weighs 1kg per meter of length.

Solution: When the piano is x meters above the ground, it is supported by $20 - x$ meters of rope which weighs $(20 - x)$ kg. The piano and the rope together weigh $100 + (20 - x) = (120 - x)$ kg, and the force of gravity is $(120 - x)g$ N (note that $N = kg \frac{m}{s^2}$). Total work required to get it to 10m above ground is, in Joules,

$$\int_0^{10} (120 - x)g \, dx = g \left(1200 - \frac{x^2}{2} \right) \Big|_0^{10} = 1150g \approx 1128.$$