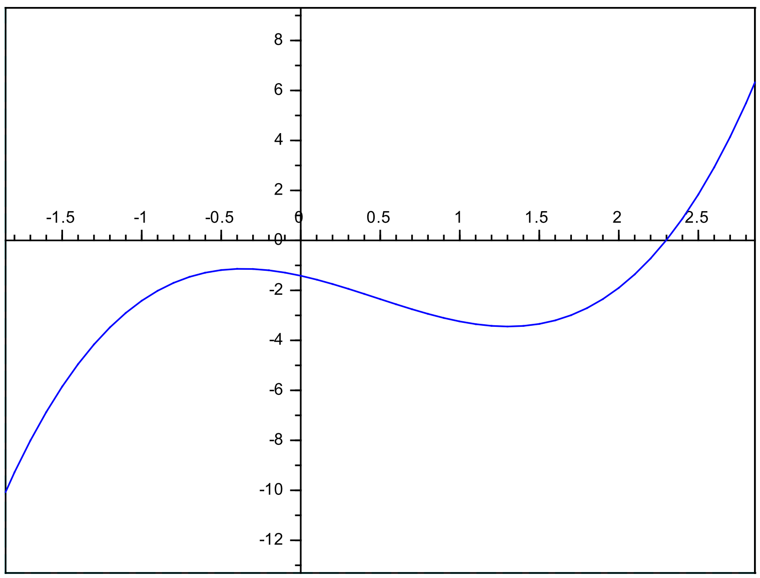
Homework problem (due Wednesday, Febrauary 26th): The function $1/(x^2 - 1)$ has a partial fraction decomposition with **two** summands:

$$\frac{1}{x^2-1} = \frac{0.5}{x-1} + \frac{-0.5}{x+1}$$

Below is a graph of the funciton $x^3 - \sqrt{2}x^2 - \sqrt{2}x - \sqrt{2}$:



Find how many summands there are in the partial fraction decomposition of

$$\frac{1}{x^3 - \sqrt{2}x^2 - \sqrt{2}x - \sqrt{2}}.$$

Show all reasoning.

Solution: we can see from the graph that the cubic polynomial $p(x) = x^3 - \sqrt{2}x^2 - \sqrt{2}x - \sqrt{2}$ has one real root. It is the only real root, since both critical points (horizontal tangents) are on the graph, and so $p(x)$ is increasing for $x > 2.5$ and decreasing for $x < -1.5$ (recall your first-semester calculus).

Therefore, the polynomial has a decomposition of the form $p(x) = (x - r)(ax^2 + bx + c)$ into two factors, with the quadratic factor being irreducible.

Proof by contradiction: if there was a factorization $ax^2+bx+c = a(x-r_1)(x-r_2)$, then r_1 and r_2 would also be roots of the cubic $p(x)$, which implies $r_1 = r_2 = r$, making $p(x) = a(x-r)^3$. But then $p(x)$ is just a scaled translation of the cubic $y = x^3$, which is not the case, since we see two critical points on the graph, whereas $y = x^3$ only has one.

We arrived at a contradiction, so the quadratic factor of $p(x)$ must be irreducible.
Since $p(x)$ has two irreducible factors, so there will be **two** summands in the partial fraction decomposition.
Note: I am accepting all solutions which argue that one root $\Rightarrow$ two irreducible factors $\Rightarrow$ two summands.