

Definition blitz! Do as many questions as you can.

Provide definitions for the following:

1. a *sequence* is an ordered infinite list (of real numbers, in this course), denoted $\{a_n\}_{n=n_0}^{\infty}$.

2. a *series* is a formal ordered sum of an infinite list of (real) numbers; a sequential formal sum of elements of a sequence, denoted $\sum_{n=n_0}^{\infty} a_n$.

3. the *sequence of partial sums* of a series $\sum_{k=1}^{\infty} a_k$ is the sequence $\{s_n\}$ whose n 'th term is the sum of the first n terms in the series:

$$s_n = \sum_{k=1}^n a_k = a_1 + a_2 + a_3 + \dots + a_n.$$

In other words, it is the sequence $a_1, a_1 + a_2, a_1 + a_2 + a_3, \dots$. It can also be defined recursively: $s_1 = a_1, s_n = s_{n-1} + a_n$.

4. what it means for a *sequence* to converge: a sequence $\{a_n\}$ converges to L if any neighborhood of L contains all but a finite number of terms of the sequence. Compare this with the definition in your book!

5. what it means for a *series* to converge: the sequence of its partial sums converges.

6. *Taylor series* for function $f(x)$ at c is the series

$$\sum_{k=0}^{\infty} \frac{f^{(k)}(x-c)^k}{k!},$$

where $f^{(k)}(x)$ is the k 'th derivative of $f(x)$.

7. *Taylor polynomial* of degree k for $f(x)$ is the polynomial

$$\sum_{k=0}^k \frac{f^{(k)}(x-c)^k}{k!},$$

which is simply the first $k+1$ terms of the Taylor series. Notice that the sum above is finite.

8. the *test for divergence* is the following test: a series $\sum_{k=1}^{\infty} a_k$ *diverges* if $\lim_{k \rightarrow \infty} a_k \neq 0$.

9. alternating series test is the following test: a series $\sum_{k=1}^{\infty} (-1)^k a_k$ *converges* if all a_k are positive, and the sequence $\{a_k\}$ is monotone converging to 0.

10. comparison test: if $\sum_{k=1}^{\infty} a_k$ is divergent and $b_k > a_k > 0$, then $\sum_{k=1}^{\infty} b_k$ is divergent.

Likewise, if $\sum_{k=1}^{\infty} a_k$ is convergent and $0 < b_k < a_k$, then $\sum_{k=1}^{\infty} b_k$ is convergent.