This quiz has 5 questions of equal value; 4 correct answers is a full score.

1. Find the area between the curve $y = -(x+4)^2$ and the x-axis for x between 2 and 5.

   Solution: Note that $f(x) = -(x+4)^2$ does not change the sign on the interval $[2,5]$. Therefore, the following integral computes the area:

$$\int_2^5 |-(x+4)^2|dx = \int_2^5 (x+4)^2 dx = \int_6^9 u^2 dx = \frac{1}{3}x^3\Big|_6^9 = 171.$$

2. Find the area of the region bounded by the curve $y = (x-3)^2$ and line $y = x-3$.

   Solution: solving $(x-3)^2 = (x-3)$ gives the solutions $x = 3, 4$. Since $(x-3)^2 < (x-3)$ on $[3,4]$, the following integral gives the area:

$$\int_3^4 (x-3) - (x-3)^2 dx = \int_0^1 u - u^2 du = \frac{u^2}{2} - \frac{u^3}{3}\Big|_0^1 = \frac{1}{6},$$

   where the integral was computed via the substitution $u = (x-3)$.

3. Find the area bounded by the curve $y = \ln x^2$, the y-axis, and the lines $y = 2$ and $y = 6$.

   Solution: we integrate on the y axis. To do this, we compute x as a function of y:

$$y = \ln x^2 \Rightarrow e^y = x^2 \Rightarrow x = e^{\frac{y}{2}}.$$

   Now $e^{\frac{y}{2}}$ does not change sign for y in $[2,6]$, so the area is given by

$$\int_2^6 e^{\frac{y}{2}} dy = 2e^{\frac{y}{2}}\Big|_2^6 = 2e^3 - 2e.$$

4. Find the volume of the solid of revolution obtained by revolving the region between the curves $y = 2\sqrt{x}$ and $y = x$ around the x-axis.

   Solution: note that $2\sqrt{x} = x$ for $x = 0, 4$ and $2\sqrt{x} > x$ on $[0,4]$. The volume of the solid of revolution is then given by the integral

$$V = \int_0^4 \pi\left((2\sqrt{x})^2 - (x)^2\right) dx = \pi\int_0^4 \pi(4x - x^2)dx = \pi(2x^2 - \frac{x^3}{3})\Big|_0^4 = \frac{32}{3}\pi.$$

5. Find the volume of a solid whose base is a circle of radius 3 and the cross-sections perpendicuar to the base are squares.

   Solution: a cricle of radius 3 can be given by an equation $x^2 + y^2 = 9$. We can pick coordinates so that the x acis runs across the cross-sections (perpendicularly to them), and y axis is parallel to the cross sections.

The volume of the solid is then given by the integral

$$V = \int_{-3}^{3} A(x)dx,$$

where $A(x)$ gives the cross-section area when the solid is cut by a plane through x perpendicular to the base and parallel to the y axis.

That is, for any x_0, $A(x_0)$ is the area of the square whose side is the intersection of the line $x = x_0$ with the circle $x^2+y^2 = 9$. Solving for y in terms of x_0 gives $y = \pm\sqrt{9 - x_0^2}$, so the side of the square has length $2\sqrt{9 - x_0^2}$, and $A(x_0) = (2\sqrt{9 - x_0^2})^2$. Therefore,

$$A(x) = 4(9 - x^2).$$

Plugging this back into the integral, we obtain

$$V = \int_{-3}^{3} 4(9 - x^2)dx = 4(9x - \frac{x^3}{3})\Big|_{-3}^{3} = 18 \cdot 8 = 144.$$