This quiz has 4 questions of value 3; 10 points is a full score.

1. Compute the length of the graph of $y = \frac{2}{3}x^{3/2}$ for $0 \leq x \leq 15$.

   Solution: Parametrize the curve: $x = t$; $y = 2/3t^{3/2}$. Then $dx = 1\ dt$, $dy = \sqrt{t}\ dt$, and $ds = \sqrt{1+t}dt$. The following integral computes the length:

$$\int_0^{15} \sqrt{1+t}\ dt = \int_1^{16} \sqrt{u}\ du = \frac{2}{3}u^{3/2}\Big|_1^{16} = \frac{2}{3}(64-1) = 42.$$

2. Compute the surface area of a sphere of radius 1 by setting up an integral of a surface of revolution and evaluating it.

   Solution: there are many solutions depending on how you parametrize the semicircle (the curve being rotated). One way is to set: $\gamma(t) = (\cos t, \sin t)$, $t = 0..\pi$ and rotate around the x-axis. Then $dx = -\sin t\ dt$, $dy = \cos t\ dt$, and

$$ds = \sqrt{dx^2 + dy^2}\ dt = \sqrt{\sin^2 t + \cos^2 t}dt = dt$$

   (such parametrization is called an *arclength* parametrization, because the change in parameter t is how far you move along the curve: this is what $ds = dt$ means).

   The distance $r(t)$ to the axis of revolution is $r(t) = y(t) = \sin t$, so the integral that gives the area is

$$A = \int_0^{\pi} 2\pi\ r\ ds = 2\pi \int_0^{\pi} \sin(t)dt = 2\pi(-\cos t)\Big|_0^{\pi} = 4\pi.$$

   Another parametrization is $x = t$, $y = \sqrt{1-t^2}$, $t = -1..1$.

3. Find the limit:

$$\lim_{x\to\infty} \frac{1}{x} \sin(x)$$

   Solution: bound by $\pm 1/x$ and use the Squeeze lemma. Specifically, observe that

$$\frac{-1}{x} \leq \frac{\sin x}{x} \leq \frac{1}{x},$$

   since $-1 \leq \sin x \leq 1$, and

$$\lim_{x\to\infty} \frac{-1}{x} = \lim_{x\to\infty} \frac{1}{x} = 0,$$

   and so $\lim_{x\to\infty} \sin x/x = 0$ by the Squeeze lemma.

4. let $x = x(t), y = y(t)$ define a parametric curve in a plane. Assume the values of x and y are given for values of $t = \{t_1, t_2, \dots, t_n\}$ and no other data is provided. Set up a sum approximating the length of the curve between $t = t_1$ and $t = t_n$.

   Solution: we are given successive points on a curve. To approximate the length, all we can do is **simply connect the dots**, and calculate the length of the broken line.

   Write $x_i = x(t_i)$, $y_i = y(t_i)$. The distance between successive points (x_i, y_i) and (x_{i+1}, y_{i+1}) is

$$d = \sqrt{(x_{i+1} - x_i)^2 + (y_{i+1} - y_i)^2},$$

   as you have learned **in your high school algebra**. The total length is simply the sum

$$\sum_{i=1}^{n-1} \sqrt{(x_{i+1} - x_i)^2 + (y_{i+1} - y_i)^2}.$$

   NOTE: regrettably, more than 90% of the class got this problem wrong. If you are one of them, it means that you don't really understand the concept of length.

   The problem, as you see, **does not even require calculus**. (It **is** a calculus problem, since this sum becomes the familiar integral in the limit).

   I have received a variety of **incorrect** solutions for this problem, including:

   - anything having an integral sign (the no-thinking solution). The problem asked for a **sum**. Also, you can't compute an integral without having $x(t)$ and $y(t)$ which were **not provided**;
   - anything involving $x'(t)$ or $y'(t)$, e.g.

$$\sum \sqrt{x'(t_i)^2 + y'(t_i)^2}(t_i - t_{i-1}) :$$

     you can't calculate this sum because neither $x(t)$ nor $y(t)$ were provided, so you don't know what $x'(t)$ or $y'(t)$ are.
   - specifically, the following sum is doubly wrong:

$$\sum \sqrt{x'(t_i)^2 + y'(t_i)^2}$$

     (even if you had $x(t)$ and $y(t)$, this would give you the wrong answer).

   Finally, I have spoken about this at length during the lab, and sent out a Matlab example which explained the concepts. If you got this problem wrong, go back and run and understand the Matlab example on length (it is posted on the TA web page).