The quiz is scored out of 10 points. Series: for the following series, determine whether the given series converges or diverges

- $\sum_{k=7}^{\infty} \frac{5}{k^{2/3}}$ **diverges**: P-test with exponent $2/3 < 1$, or by comparison:

$$\sum_{k=7}^{\infty} \frac{5}{k^{2/3}} > 5 \sum_{k=7}^{\infty} \frac{1}{k} = \infty.$$

- $\sum_{k=7}^{\infty} \frac{5k^3 + 3k}{k!}$ **converges**. The idea is that $k!$ grows faster than any polynomial. In particular, $k! > k^6 > (5k^3 + 3k)k^2$ if $k > 12$, so

$$\sum_{k=7}^{\infty} \frac{5k^3 + 3k}{k!} = C + \sum_{k=12}^{\infty} \frac{5k^3 + 3k}{k!} < \sum_{k=12}^{\infty} \frac{5k^3 + 3k}{(5k^3 + 3k)k^2} = \sum_{k=12}^{\infty} \frac{1}{k^2},$$

which converges. Therefore, our series converges by the comparison test.

- $\sum_{k=7}^{\infty} k \sin\left(\frac{1}{k}\right)$ **diverges**. Indeed,

$$\lim_{k\to\infty} k \sin\left(\frac{1}{k}\right) = \lim_{k\to\infty} \frac{\sin\left(\frac{1}{k}\right)}{1/k} = \lim_{u\to 0} \frac{\sin(u)}{u} = 1$$

(by setting $u = 1/k$). The series therefore diverges by the divergence test.

- $\sum_{k=7}^{\infty} \frac{k^2 + 5}{3k^3 + 1}$ **diverges**: asympotically, $\frac{k^2 + 5}{3k^3 + 1} \approx \frac{1}{3k}$, and so

$$\lim_{k\to\infty} \frac{k^2 + 5}{3k^3 + 1} \Big/ \left(\frac{1}{3k}\right) = \lim_{k\to\infty} \frac{1 + 5/k^2}{3k + 1/k^2} \Big/ \left(\frac{1}{3k}\right)$$
$$= \lim_{k\to\infty} \frac{3k}{3k} = 1,$$

and $\sum_{k=7}^{\infty} \frac{1}{3k}$ diverges. Therefore, our series diverges by the limit comparison test.

- $\sum_{k=7}^{\infty} \frac{\sin(2^k) + 1}{2^k + 1}$ **converges**:

$$0 \leq \frac{\sin(2^k) + 1}{2^k + 1} < \frac{2}{2^k + 1} < 2\frac{1}{2^k},$$

and $\sum_{k=7}^{\infty} \frac{1}{2^k}$ converges. So our series converges by the comparison test.

- Given a series $\sum_{k=1}^{\infty} a_k$, define a sequence $\{b_n\}$ such that the series converges if an only if the sequence $\{b_n\}$ does.

  Solution: this question tests your understanding of the meaning of convergence of series. The answer is, verbatim, the definition of what it means for the series to converge. Read your textbook!

  Spefically, a series is said to converge when the **sequence of partial sums**, defined by

  $$b_n = \sum_{k=1}^{n} a_k,$$

  converges.